

# Conceptual Discussion on Free Vibration Analysis of Tension Leg Platforms

Mohammad Reza Tabeshpour

Center of Excellence in Hydrodynamics and Dynamics of Marine Vehicles, Mechanical Engineering Department, Sharif University of Technology, Tehran, Iran

tabeshpour@sharif.edu

## Abstract

Tension leg platforms are used for oil exploitation in deep water. The accurate and reliable responses are needed for optimum design and control of the structure. Free vibration analysis gives us a deep view of structural response and the judgment on coupling effects between degrees of freedom. In this paper the nonlinear dynamic analysis of TLP is carried out. The power spectral densities (PSDs) of responses are calculated from nonlinear responses. A computer program has been developed capable of solving differential equation of motion of TLP under various types of excitations. The focus of the paper is on the conceptual discussion of the structural behavior and responses of the structure related to initial arbitrary conditions. As an example a case study is investigated and numerical results are discussed.

## Keywords

TLP; Free Vibration; Dynamic Analysis

## Introduction

It is obvious that there is an increasing demand for oil exploitation in deep water. As the water depth increase, the environment will be severer and therefore some innovative structures are required for economic production of gas and petroleum in deep water. There are several systems that have been developed for deep water. Because of high stresses if fixed structures used in deep water, deep water systems will be compliant and therefore these structures must be designed dynamically. One of the important types of these structures is TLP (Fig 1)

Many studies have been carried out to understand the structural behavior of TLP and determine the effect of several parameters on dynamic response and average life time of the structure (Faltinsen et al., 1982; Angelides, et al., 1982; Teigen 1983; Ahmad, 1996; Jain, 1997; Chandrasekaran and Jain, 2002; Tabeshpour et al., 2006a). The tether system is a critical and basic component of the TLP. The most important point in the design of TLP is the pretension of the legs. The

pretension causes that the platform behaves like a stiff structure with respect to the vertical degrees of freedom (heave, pitch and roll), whereas with respect to the horizontal degrees of freedom (surge, sway and yaw) it behaves as a floating structure. Therefore the periods of the vertical degrees of freedom are lower than that of the others.

Another important task is to investigate the effects of radiation and scattering on the hull and tendon responses. An analytical solution to surge motion of TLP has been proposed and demonstrated (Lee et al., 1999), in which the surge motion of a platform with pre-tensioned tethers was calculated. In that study, however, the elasticity of tethers was only implied and the motion of tethers was also simplified as on-line rigid-body motion proportional to the top platform. Thus, both the material property and the mechanical behavior for the tether incorporated in the TLP system were ignored.

An important point in that study is linearization of the surge motion. But it is obvious that the structural behavior in the surge motion is highly nonlinear because of large deformation of TLP in the surge motion degree of freedom (geometric nonlinearity) and nonlinear drag forces of Morison equation. Therefore the obtained solution is not suitable for the actual engineering application. For heave degree of freedom the structural behavior is linear, because there is no geometric nonlinearity in the heave motion degree of freedom and drag forces on legs have no vertical component. Similarly, an analytical heave vibration of TLP with radiation and scattering effects for damped systems has been presented (Tabeshpour et al., 2006b). A similar method is presented for hydrodynamic pitch response of the structure (Tabeshpour et al., 2006c). The modified Euler method (Karman and Biot, 1940) presented herein is a simple numerical procedure which can be effectively used for the analysis of the dynamic response of structures in

the time domain. It has been shown that the modified Euler method is conditionally stable (Hahn, 1990).

The application of the modified Euler method made herein shows that it is efficient and easy to use, and that it can be employed to obtain accurate solutions to a wide variety of structural dynamics problems. Simplicity is one of the distinguishing features of the method. Because the modified Euler method is conditionally stable, it may be inefficient for the analysis by means of direct integration of the response of a multi degree-of-freedom system with a very short but the highest natural period of vibration. However, the method is explicit, and particularly suitable for the analysis of non-linear systems. The modified Euler method has been successfully used in the analysis for the dynamic response of wave-excited offshore structures (Sanghvi, 1990).

The effect of added mass fluctuation on the heave response of tension leg platform has been investigated by using perturbation method both for discrete and continuous models (Tabeshpour et al., 2006d).

Liu et al. described an analysis of the non-linear effects and identification of non-linear pitch motion on tension leg platforms. The purpose of their paper was to accurately identify pitch motion on the tension leg platform and to interpret the non-linear effects using statistical methods, the NARMAX methodology, and the higher order frequency response functions (Liu et al., 2004).

A robust stochastic design framework has been discussed for design of mass dampers by Taflanidis et al. (2009) was focused on applications for the mitigation of the coupled heave and pitch response of Tension Leg Platforms under stochastic sea excitation.

Tabeshpour et al. (2010) investigated design and effect of tuned mass damper on response of tension leg platform under wind and wave forces. Efficiency of numerical methods is a priority to dynamic nonlinear analyses. It is possible to use numerical methods such as beta-Newmark in order to investigate the structural response behavior of the dynamic systems under random sea wave loads but because of necessity to analyze the offshore systems for extensive time to fatigue study, it is important to use simple stable methods for numerical integration. The key point of suitability of MEM to solve the TLP system is that the maximum frequency of the system is about 0.5 Hz. Tabeshpour et al. (2006e) has investigated the efficiency of the modified Euler method (MEM) for

nonlinear dynamic analysis of tension leg platforms as a response dependent system.

Convergence and stability analyses of MEM were presented by Tabeshpour et al. (2012) Modified Euler method (MEM) has been used for solving the differential equation of. Surge motion analysis of TLP under linear wave via perturbation method has been presented by Tabeshpour and Shoghi (2011) who has also been presented limitation between linear and nonlinear models for surge motion of TLP.

In this paper an integrated response analysis of TLP under arbitrary initial values is presented giving a deep view of results. Displacements and strains of tendons are investigated and discussed in this paper under initial arbitrary independent values. Modified Euler method is used to solve nonlinear differential equation of motion. Phase plane used in here is a helpful tool to discuss the stability and convergence of the response.

## Structural Modelling of TLP

### Mass Matrix

Structural mass is assumed to be lumped at each degree of freedom. Hence, it is constant and diagonal in nature. The added mass,  $M_a$ , due to the water surrounding the structural members and arising from the modified Morrison equation, is considered up to the mean sea level (MSL) only. The fluctuating component of added mass due to the variable submergence of the structure in water is considered in the force vector depending upon whether the sea surface elevation is above (or) below the MSL. The mass matrix of TLP is

$$[M] = \begin{bmatrix} \text{Surge} & \text{Sway} & \text{Heave} & \text{Roll} & \text{Pitch} & \text{Yaw} \\ M'_{ss} & 0 & 0 & 0 & 0 & 0 \\ 0 & M'_{ww} & 0 & 0 & 0 & 0 \\ 0 & 0 & M'_{hh} & 0 & 0 & 0 \\ M_{aRS} & M_{aRW} & M_{aRH} & M_{RR} & 0 & 0 \\ M_{aPS} & M_{aPW} & M_{aPH} & 0 & M_{PP} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{YY} \end{bmatrix} \quad (1)$$

where  $M_{ss} = M_{ww} = M_{hh} = M$  ,  $M'_{ss} = M_{ss} + M_{ass}$  ,  $M'_{ww} = M_{ww} + M_{aww}$  and  $M'_{hh} = M_{hh} + M_{ahh}$  .  $M$  is the total mass of the entire structure,  $M_{RR}$  is the total mass moment of inertia about the  $x$  axis =  $Mr_x^2$  ,  $M_{PP}$  is the total mass moment of inertia of the  $y$  axis =  $Mr_y^2$  ,  $M_{YY}$  is the total mass moment of inertia of the  $z$  axis =

$Mr_z^2$ ,  $r_x$  is the radius of gyration of the  $x$  axis,  $r_y$  is the radius of gyration of the  $y$  axis, and  $r_z$  is the radius of gyration of the  $z$  axis. The added mass terms are:

$$M_{ass} = M_{awW} = M_{aHH} = 0.25\pi D^2(C_m - 1)\rho dl \quad (2)$$

$$M_{ass} = \int_{length} dM_{ass} \quad (3)$$

$M_{aRS}$ ,  $M_{aRW}$  and  $M_{aRH}$  are the added mass moment of inertia in the roll degree of freedom due to hydrodynamic force in the surge, sway and heave direction, respectively.  $M_{aPS}$ ,  $M_{aPW}$ , and  $M_{aPH}$  are the added mass moment of inertia in the pitch degree of freedom due to hydrodynamic force in the surge, sway and heave direction, respectively. The presence of off diagonal terms in the mass matrix indicates a contribution to the added mass due to the hydrodynamic loading. The loading will be attracted only in the surge, heave and pitch degrees of freedom due to the unidirectional wave acting in the surge direction on a symmetric configuration of the platform about the  $x$  and  $z$  axes).

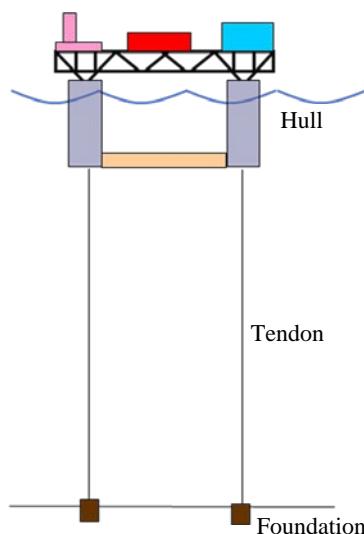


FIG. 1 TLP CONFIGURATION

### **Stiffness Matrix of the TLP**

The coefficients,  $K_{AB}$ , of the stiffness matrix of the triangular TLP are derived as the reaction in the degree of freedom A due to unit displacement in the degree of freedom B, keeping all other degrees of freedom restrained. The coefficients of the stiffness matrix have nonlinear terms due to the cosine, sine, square root and squared terms of the displacements. Furthermore, changes in tendon tension due to the

motion of the TLP in different degrees of freedom make the stiffness matrix response-dependent. The stiffness matrix  $[K]$  of a TLP is:

$$[K] = \begin{bmatrix} Surge & Sway & Heave & Roll & Pitch & Yaw & \\ \begin{matrix} K_{ss} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{ww} & 0 & 0 & 0 & 0 \\ K_{hs} & K_{hw} & K_{hh} & K_{hr} & K_{hp} & K_{hy} \\ 0 & K_{rw} & 0 & K_{rr} & 0 & 0 \\ K_{ps} & 0 & 0 & 0 & K_{pp} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{yy} \end{matrix} & \begin{matrix} Surge \\ Sway \\ Heave \\ Roll \\ Pitch \\ Yaw \end{matrix} \end{bmatrix} \quad (4)$$

$S$ ,  $W$ ,  $H$ ,  $R$ ,  $P$  and  $Y$  denote surge, sway, heave, roll, pitch and yaw respectively. These elements have been investigated by many researchers [2-7].

In the stiffness matrix, the presence of off-diagonal terms reflects the coupling effect between the various degrees of freedom and the coefficients depend on the change in the tension of the tendons, which affects the buoyancy of the system. Hence, the  $[K]$  is not constant for all time instants but the coefficients are replaced by a new value computed at each time instant depending upon the response value at that time instant.

### **Damping Matrix, $[C]$**

Assuming  $[C]$  to be proportional to  $[K]$  and  $[M]$ , the elements of  $[C]$  are determined by the equation given below:

$$C = \alpha M + \beta K \quad (5)$$

$\alpha$  and  $\beta$  are constant. This matrix is calculated based on the initial values of  $[K]$  and  $[M]$  only. There are two approaches to define  $\alpha$  and  $\beta$ . The first method to calculate them in a manner that a predefined damping ratio  $\xi = C / 2M \omega$  is achieved in which  $\omega$  is angular frequency of the system. The second method is to consider an appropriate value for them such as 2%-5%.

### **Equation of Motion**

The free vibration of equation of motion of the TLP is given as:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F(t)\} \quad (6-a)$$

$$\{F(t)\} = \left\{ \int 0.5\rho_w C_d D \dot{x} |\dot{x}| + (0.25\pi D^2 [C_m - 1]\rho_w \dot{x}) \right\} \quad (6-b)$$

where  $[M]$ ,  $[C]$  and  $[K]$  are the matrices of mass, damping and stiffness respectively,  $\{X\}$ ,  $\{\dot{X}\}$  and  $\{\ddot{X}\}$  are the structural displacement, velocity and

acceleration vector respectively and  $\{F(t)\}$  is the excitation force vector.

### Case Study

A TLP in 800m deep water has been chosen for the numerical analysis. The characteristics of the TLP are shown in table 1. Free vibration analysis is carried out using a MATLAB based code developed for nonlinear dynamic analysis of TLP.

TABLE 1 TLP PROPERTIES

Platform height	80 m
Platform weight	$1.3 \times 10^9 N$
Total pretension	$0.8 \times 10^9 N$
Draft	45.28 m
Corner column diameter	18 m
Center to center column spacing	90 m
Pontoon length	90-18 m
Pontoon diameter	12 m
Center of gravity above the base line	10 m
Tether length	800-45.28 m
Diameter of tether pipe	1.2 m
Youngs modules of tether material	$2.019 \times 10^{11} N / m^2$
Number of columns	4
Service life	20 years
Radii of gyration ( $R_x, R_y, R_z$ )	55 m, 55 m and 65 m
Drag coefficient, $C_D$	1.0
Inertia coefficient, $C_m$	1.8
Mass density of water	1025 kg / m <sup>3</sup>

Draft is calculated as follows:

$$D_r = \frac{\left[ \left\{ \frac{W+T}{\rho \pi g} \right\} - D^2 S \right]}{D_c^2}$$

where  $W$  is the platform weight,  $T$  is pretension of tendons,  $\rho$  is water density,  $D_c$  is diameter of column,  $D$  is pontoon diameter,  $S$  is distance between pontoons.

Initial values assigned for free vibration of TLP are shown in table 2. Because the stiffness is related to displacement and initial displacement of surge motion (30m) is more than sway (20m), therefore surge stiffness is more than sway stiffness and surge period

(63.39s) is lower than that of sway (76.67s). Table 3 shows TLP periods and frequencies for all degrees of freedom.

TABLE 2 TLP PROPERTIES

Motion	Initial Value
surge	30 m
sway	20 m
Heave	0.5 m
Roll	0.01 rad
Pitch	0.01 rad
Yaw	0.01 rad

TABLE 3 TLP PERIODS AND FREQUENCIES

Motion	Period (sec)	Frequency (rad/s)
surge	63.39	0.1
sway	76.67	0.08
Heave	1.98	3.17
Roll	1.74	3.61
Pitch	1.74	3.61
Yaw	80.57	0.078

Fig 2 shows the nonlinear relation between the forces in surge direction and surge displacement. The system is hardening and geometry nonlinearity is important to increase the stiffness in large displacements. Surge stiffness is as follow:

$$K_{ss} = \frac{n(T_0 + \sqrt{x^2 + l^2} - l)AE / l}{\sqrt{x^2 + l^2}}$$

$$= \frac{nT_0}{l\sqrt{\frac{x^2}{l^2} + 1}} + nT_0AE \times \frac{\sqrt{\frac{x^2}{l^2} + 1} - 1}{l\sqrt{\frac{x^2}{l^2} + 1}}$$

$$= \frac{nT_0}{l\left(1 + \frac{x^2}{2l^2}\right)} + \frac{nT_0AE}{l\left(1 + \frac{x^2}{2l^2}\right)} \frac{x^2}{2l^2} \approx \frac{nT_0}{l} + \frac{nT_0AE}{2l} \frac{x^2}{l^2}$$

It means that the stiffness is a parabolic function.

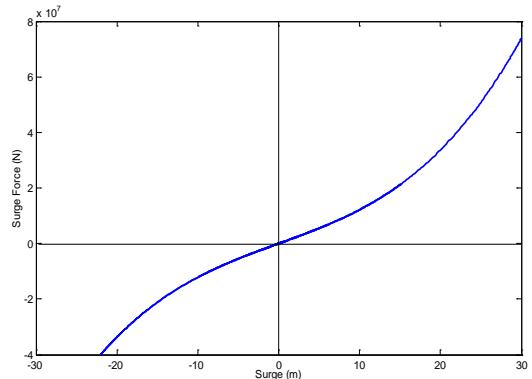


FIG. 2 RELATION BETWEEN SURGE FORCE AND SURGE DISPLACEMENT

Surge stiffness has two parts:

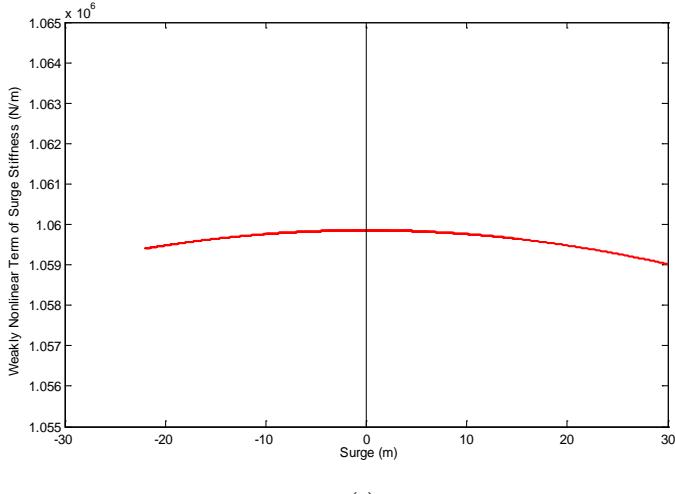
- Weakly nonlinear part:  $\frac{nT_0}{l \left(1 + \frac{x^2}{2l^2}\right)}$

This part is approximately linear.

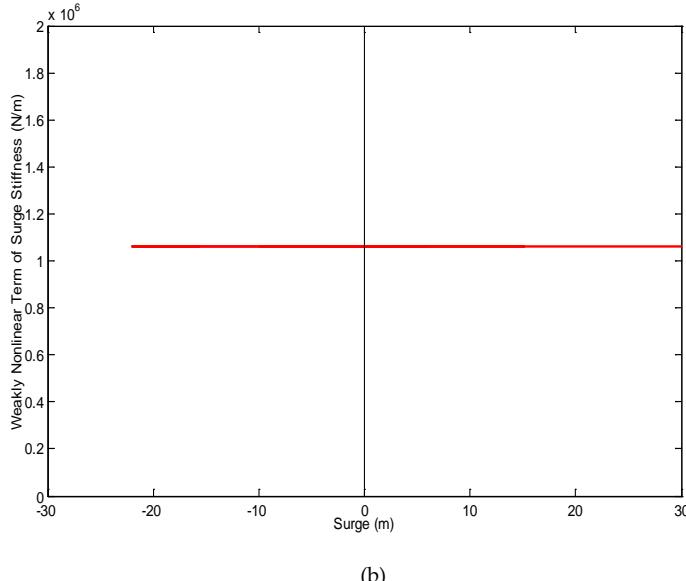
- Strongly nonlinear part:  $\frac{nT_0 AE}{l \left(1 + \frac{x^2}{2l^2}\right)} \frac{x^2}{2l^2}$

This part is parabolic. Figs 3 and 4 show weakly and strongly nonlinear parts of surge stiffness respectively.

Fig 5 shows the surge stiffness versus surge motion. It is seen that in approximately 3% relative displacement (surge=27m) stiffness is becoming more than two times comparing zero displacement.



(a)



(b)

FIG. 3 WEEKLY NONLINEAR PART OF SURGE STIFFNESS

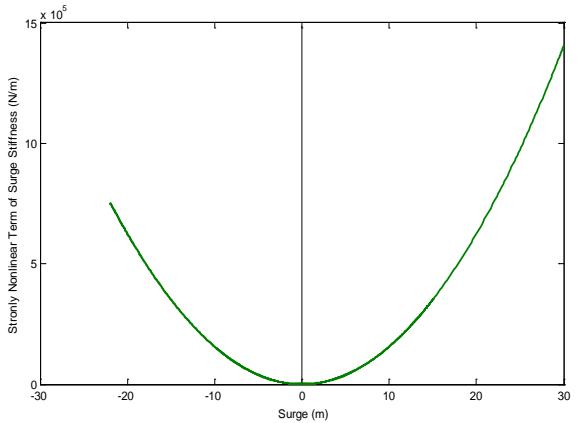


FIG. 4 STRONGLY NONLINEAR PART OF SURGE STIFFNESS

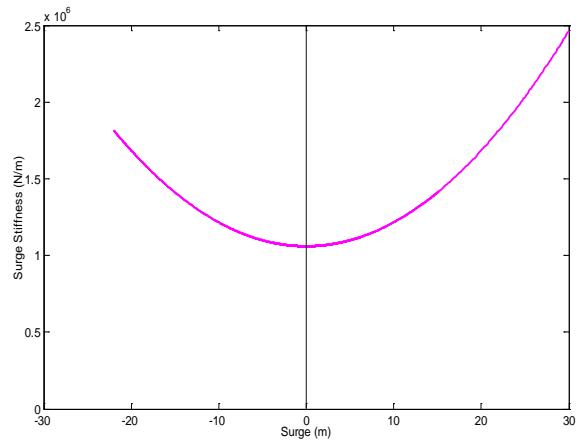


FIG. 5 SURGE STIFFNESS VERSUS SURGE MOTION

Fig 6 shows the variation of surge stiffness versus time. Comparing weak and strong parts of stiffness shows the importance of strong part in large displacements. The total varying stiffness in time will reduce to a minimum value for each cycle with two rounds.

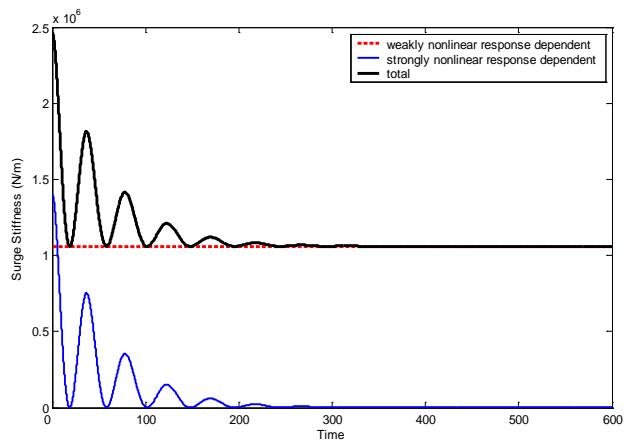


FIG. 6 VARIATION OF SURGE STIFFNESS VERSUS TIME

Fig 7 shows time histories of responses. It is seen that all degrees of freedom start from initial dictated value to the zero point, but there is a short time of instability in heave motion because of the absence of unison and

compatibility among the arbitrary initial values. However the response converges to the zero equilibrium point.

Heave response of left hand corner of TLP is shown in Fig 8. Transient response at the first three seconds shows a temporary instability of the response due to the exclusion of compatibility among initial values. However the response converges because the system is hardening.

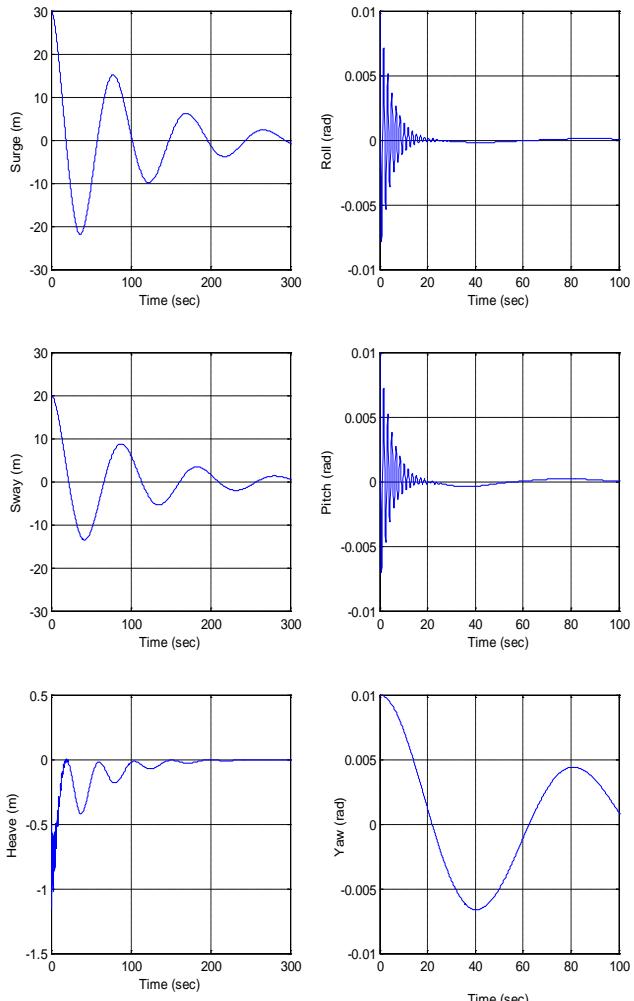
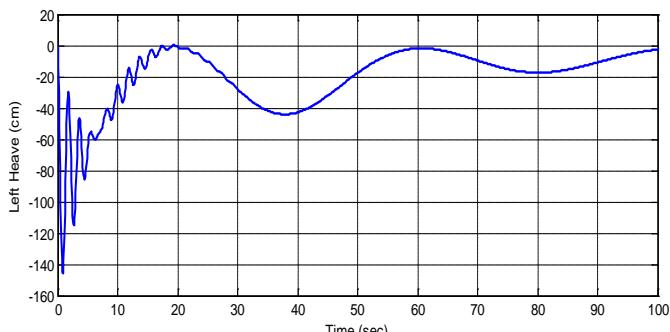
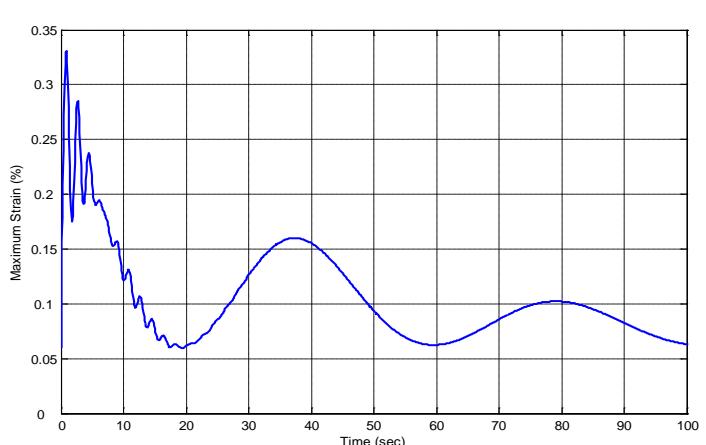
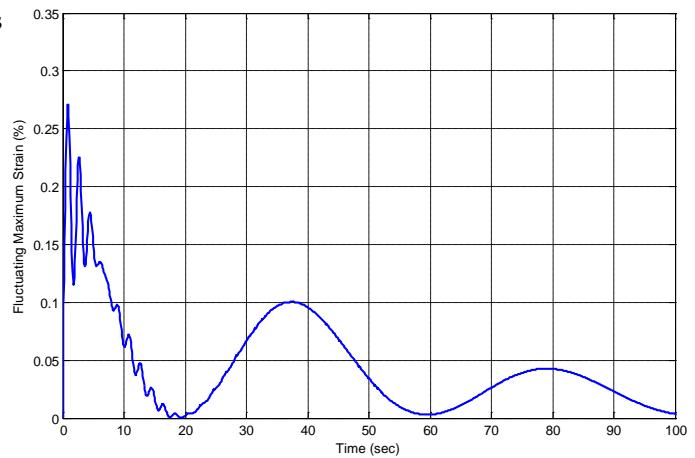


FIG. 7 TIME HISTORIES OF RESPONSES



There is an initial strain in tendons because of pretension stress. Therefore when tendons are under tension in nonzero condition, the strain can be divided into two parts: initial strain and fluctuating strain. Fluctuating part of strain in right hand side tendon is shown in Fig 9. Similarly Fig 10 shows total strain in left hand side tendon.



Trajectory of TLP motion in space gives a global view of the dynamic response and the path of the system wending from the start point to zero point (Fig 11).

A horizontal view of TLP motion is shown in Fig 12. the convergence from the start point (30, 20) to the zero equilibrium point can be observed clearly.

Phase planes of all degrees of freedom show converging to unique equilibrium stable point. There is a special case in heave motion. Firstly the initial point is approaching from the initial point (1) to point 5 that has the largest distance from the initial point. This event repeats for some other cycles which happen

because the initial values are arbitrary and independent to each other. Then the point is approaching to the zero equilibrium point on the spiral path.

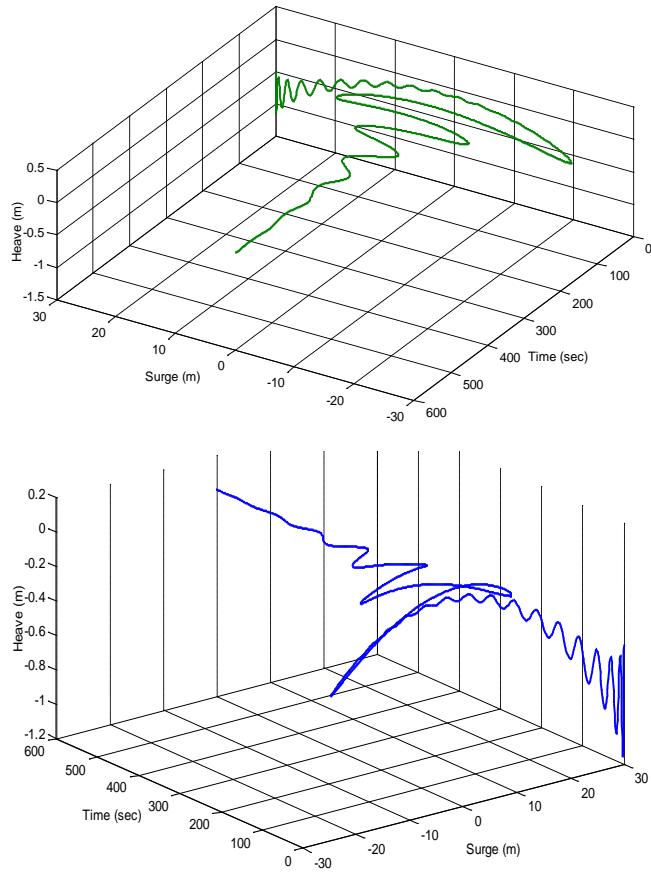


FIG. 11 TRAJECTORY OF TLP MOTION IN SPACE

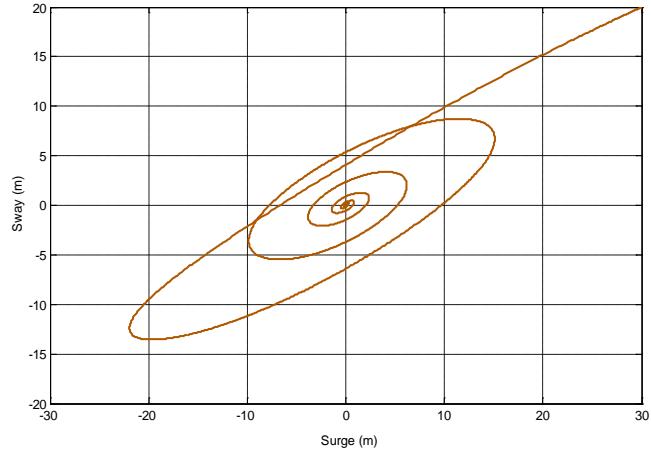


FIG. 12 HORIZONTAL VIEW OF TLP MOTION

Power spectral densities of sway and pitch responses are shown in Figs 14 and 15. Because of coupling, there are two peaks in some degrees of freedom. Peaks are occurred at the related frequency of each degree of freedom.

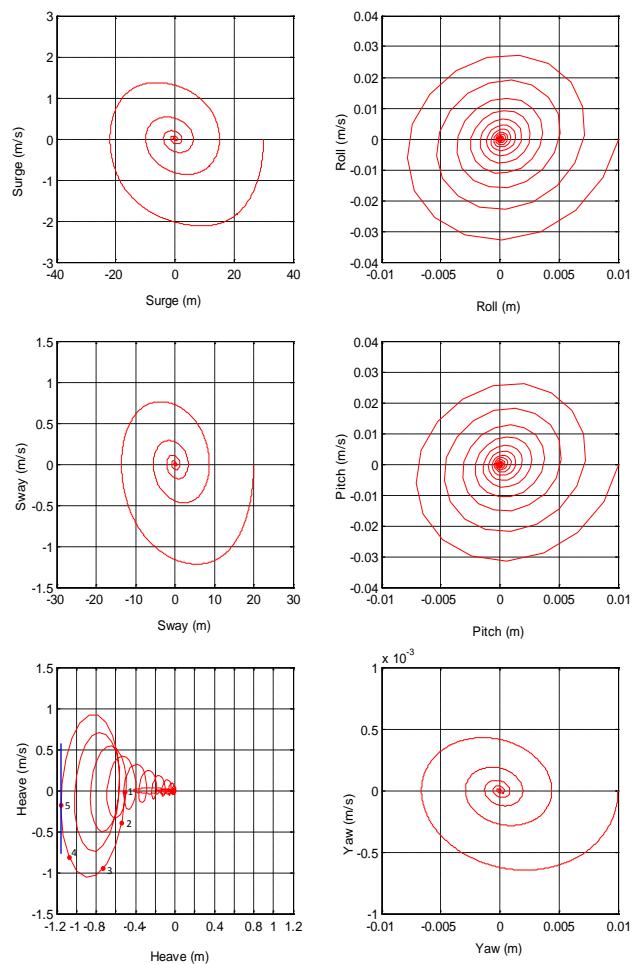


FIG. 13 PHASE PLANES

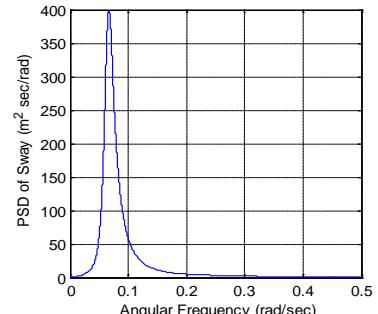


FIG. 14 POWER SPECTRAL DENSITY OF SWAY

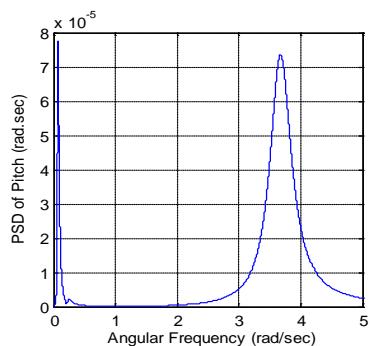


FIG. 15 POWER SPECTRAL DENSITY OF PITCH

## Conclusions

A conceptual discussion on the results of nonlinear dynamic analysis of TLP under arbitrary initial values has been presented. A special case has been examined at the initial transition part of heave motion diverging from equilibrium point. Because the system is hardening, this incompatible initial value is free from diverging problem. Modified Euler method suitable method for nonlinear dynamic analysis of response dependent systems was used to solve nonlinear differential equation of motion. The power spectral densities (PSDs) of responses were calculated from nonlinear responses.

## REFERENCES

Angelides, DC, Chen, C., Will, SA, 1982. Dynamic response of tension leg platform. Proceedings of BOSS 2, 100-120.

Ahmad S. Stochastic TLP response under long crested random sea. J Comp Struct 1996;61(6):975-93.

Chandrasekaran S, Jain AK. Triangular configuration tension leg platform behavior under random sea wave loads. Ocean Eng 2002; 29(15): 1895-928.

Faltinsen OM, Van Hooff RW, Fylling IJ, Teigen PS. Theoretical and experimental investigations of tension leg platform behavior. In: Proceedings of BOSS 1, 1982. p. 411-23.

Hahn, GD, (1990). "A Modified Euler method for dynamic analysis," International Journal for Numerical Methods in Engineering, Vol 32, No 3, pp 943-955.

Jain AK. Nonlinear coupled response of offshore tension leg platforms to regular wave forces. Ocean Eng 1997;24(7):577-92.

Karman, T. V., and Biot, M. A., (1940). "Mathematical in Engineering," McGraw-Hill, NewYork.

Lee HH, Wang P-W, Lee C-P. Dragged surge motion of tension leg platforms and strained elastic tethers. Ocean Eng 1999;26(2):579-94.

Liu J-J, Huang Y-F, and Lin H-W, "Nonlinear pitch motion identification and interpretation of a tension leg platform", Journal of Marine Science and Technology, Vol. 12, No. 4, pp. 309-318 (2004).

Sanghvi, JR, (1990). "Simplified dynamic analysis of offshore structures," M.S. Thesis, Department of Civil and Environmental Engineering, Vanderbilt University.

Tabeshpour, M. R., Golafshani, A. A., and Seif, M. S., (2006a), "A Comprehensive study on the results of tension leg platform responses in random sea", Journal of Zhejiang University, Vol. 7, No. 8, pp. 1305-1317.

Tabeshpour MR, Golafshani AA, Ataie Ashtiani B, Seif MS. Analytical solution of heave vibration of tension leg platform. Int J Hydrol Hydromechanics 2006b;54(3):280-8.

Tabeshpour MR, Ataie Ashtiani B, Seif MS, Golafshani AA. Wave interaction pitch response of tension leg structures. In: 13th international congress on sound and vibration, Austria, Vienna; 2006c.

Tabeshpour, MR, Golafshani, AA, and Seif, MS (2006d) "Second order perturbation added mass fluctuation on vertical vibration of TLP ", Marine Structures, 19 (4), 271-283.

Tabeshpour, MR, dekharghanian, V, and Dovlatshahi, M., (2010) "The effect of tuned mass damper on vertical vibration of TLP", 12th National Conference of offshore industries.

Tabeshpour, MR, Golafshani, AA, Seif, MS, (2006e). Efficient Numerical Dynamic Analysis of Tension Leg Platforms under Sea Wave Loads, Journal of Maritime and Technology, 3(5), 1E-15E.

Tabeshpour, MR, Golafshani, AA, Seif, MS, (2012). Stability of the Modified Euler Method for Nonlinear Dynamic Analysis of TLP, International Journal of Maritime and Technology, Accepted.

Tabeshpour, MR, and Shoghi, R., (2011), Surge motion analysis of TLP under linear wave via perturbation method, Offshore industries conference, Iran (in Farsi).

Taflanidis, D. C. Angelides and J. T. Scruggs, "Simulation-based robust design of mass dampers for response mitigation of tension leg platforms", Engineering Structures, Volume 31, Issue 4, April 2009, Pages 847-857.

Teigen, PS. The response of a tension leg platform in short-crested waves. In: Proceedings of the offshore technology conference, OTC No. 4642, 1983. p. 525-32.